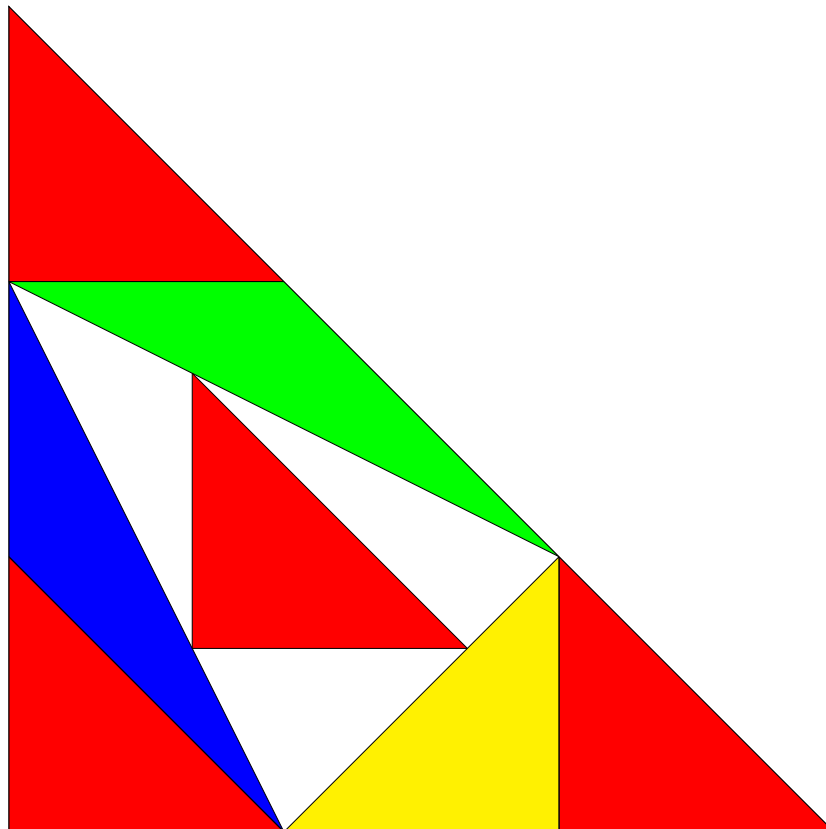




Collaborative Research Centre TRR 191

# **Symplectic Structures in Geometry, Algebra and Dynamics**

Universität zu Köln  
Ruhr-Universität Bochum



First Funding Period  
2017 – 2020

## **Call for Applications**

### **8 postdoctoral and 17 Ph.D. student positions**

The Deutsche Forschungsgemeinschaft (German Research Foundation) has granted the funding of a Collaborative Research Centre / Transregio (CRC/TRR 191) on “Symplectic Structures in Geometry, Algebra and Dynamics”. This CRC will be based at the universities of Bochum and Cologne, and it includes mathematicians from the university of Münster.

Within this CRC, 18 individual projects will be funded. We invite applications for 8 full-time postdoctoral positions (pd) and 17 positions for doctoral students (ds). Payment is based on the German TVL-13 scale if terms and conditions under collective bargaining laws are fulfilled; Ph.D. students will be given 75% positions.

The CRC provides an excellent infrastructure for training and research in an internationally visible network, and we offer intensive support for international students and postdocs. There are specially dedicated funds to support researchers with children. The positions will be based at the universities of the principal investigators of the relevant project. Many of the projects are based at more than one university.

For the postdoctoral positions we are seeking candidates holding a very good Ph.D. in one of the areas relevant to the particular project. For the doctoral student positions we are seeking highly qualified candidates holding an M.Sc. or equivalent degree with a background suitable for one of the projects. Applicants should be fluent in written and spoken German or English, a good proficiency in English is expected of all candidates.

Project summaries with the number of positions per project follow below. Candidates should specify the project(s) to which they apply.

The participating universities are equal opportunity employers. Applications by women are especially encouraged. Applications of disabled persons will be given preferential treatment to those of other candidates with equal qualifications.

The first funding period of the CRC starts in January 2017. To guarantee consideration in the first round of hiring, **applications** should reach us by **6 January 2017**. The application material (covering letter, cv, relevant degree certificates) should be sent as **a single pdf file**, the thesis of your highest degree as a further pdf file. One or two letters of reference should be sent directly by the referees.

Please send your application to Mrs Sabine Broeders ([broeders@math.uni-koeln.de](mailto:broeders@math.uni-koeln.de)).

For further information, please visit our websites

<http://www.ruhr-uni-bochum.de/ffm/index.html.en>

<http://www.mi.uni-koeln.de/home-institut/Alle.en.html>

<http://wwwmath.uni-muenster.de/42/en/institute/mathematical-institute/>

and the individual websites of the principal investigators.

# SFB/TRR 191

## Symplectic structures in geometry, algebra and dynamics

### Summary of the research programme:

Since their inception, the study of symplectic structures and the applications of symplectic techniques (as well as their odd-dimensional contact geometric counterparts) have benefited from a strong extraneous motivation. Symplectic concepts have been developed to solve problems in other fields that have resisted more traditional approaches, or they have been used to provide alternative and often conceptually simpler or unifying arguments for known results. Outstanding examples are property P for knots, Cerf's theorem on diffeomorphisms of the 3-sphere, and the theorem of Lyusternik-Fet on periodic geodesics.

The aim of the CRC is to bring together, on the one hand, mathematicians who have been socialized in symplectic geometry and, on the other, scientists working in areas that have proved important for the cross-fertilization of ideas with symplectic geometry, notably dynamics and algebra. In addition, the CRC intends to explore connections with fields where, so far, the potential of the symplectic viewpoint has not been fully realized or, conversely, which can contribute new methodology to the study of symplectic questions (e.g. optimization, computer science).

The CRC bundles symplectic expertise that will allow us to make substantive progress on some of the driving conjectures in the field, such as the Weinstein conjecture on the existence of periodic Reeb orbits, or the Viterbo conjecture on a volume bound for the symplectic capacity of compact convex domains in  $\mathbb{R}^{2n}$ . The latter can be formulated as a problem in systolic geometry and is related to the Mahler conjecture in convex geometry.

The focus on symplectic structures and techniques will provide coherence to what is in effect a group of mathematicians with a wide spectrum of interests.

Symbols:      pd      = postdoc  
                 ds      = doctoral student  
  
                 RUB     = Ruhr-Universität Bochum  
                 UzK     = Universität zu Köln  
                 WWU     = Westfälische Wilhelms-Universität Münster

## Individual projects:

### A Topology and equivariant theories

#### A1 Topological aspects of symplectic manifolds with symmetries

Heinzner, Reineke, Sabatini

1 ds (RUB), 1 ds (UzK)

We study topological properties of symplectic manifolds with a Lie group action by symplectomorphisms. Central goals are the computation of topological invariants, classifications of suitable classes of such manifolds, and approaches to symplectic analogues of the Mukai conjecture and the Kobayashi-Ochiai theorem. We study symplectic reductions of Hamiltonian actions on Kähler manifolds related to quivers, with the goal of establishing new correspondences between Gromov-Witten invariants and Donaldson-Thomas invariants of quiver moduli.

#### A2 Riemannian orbifolds

Geiges, Lytchak, Zehmisch

1 ds (UzK)

We intend to prove the existence of at least one closed geodesic on every compact Riemannian orbifold, using methods developed in symplectic topology for proving the Weinstein conjecture. We also study orbifolds all of whose geodesics are closed with the help of holomorphic curve techniques. A further aim is to understand the topology of highly connected orbifolds, with applications to Riemannian foliations on manifolds.

#### A5 Reeb dynamics and topology

Albers, Geiges, Zehmisch

1 pd (UzK), 1 ds (WWU)

The focus of this project lies on surgical constructions in contact topology. We investigate the effect of surgery on the existence of periodic Reeb orbits and the existence of positive loops of contactomorphisms. One aim is to prove the Weinstein conjecture for manifolds obtained by subcritical surgery. Further, we want to construct local models for the Reeb flow whose insertion into a given contact manifold allows us to modify the Reeb dynamics in a controlled fashion.

#### A6 Rabinowitz Floer homology

Abbondandolo, Albers

1 pd (RUB), 1 ds (WWU)

Rabinowitz Floer homology (RFH) is a powerful algebraic invariant of contact-type compact hypersurfaces inside symplectic manifolds. The aim of this project is to develop RFH further, with an eye towards applications. In particular, we intend to study relations of RFH to other Floer homologies,

product structures on RFH, equivariant versions and the behaviour of RFH under surgery constructions. We expect to derive applications about multiplicity and linear stability of periodic orbits of Reeb flows and translated points of contactomorphisms, orderability of contact manifolds, lower bounds on the complexity of Reeb flows and positive contactomorphisms.

## A7 Derived categories of singular curves

Burban, Marinescu

1 ds (UzK)

In this project, we shall apply techniques of algebraic geometry and homological algebra (derived categories, Fourier-Mukai transforms, vector bundles on possibly singular Riemann surfaces) to study problems of geometric analysis. In particular, we shall investigate Bochner Laplacians and kernel functions (Bergman and Szegő kernels) attached to vector bundles on (possibly singular) compact Riemann surfaces. Matrix-valued Szegő kernels "geometrize" the theory of the associative and classical Yang-Baxter equations. The study of Bochner Laplacians and Bergman kernels attached to line bundles on singular Riemann surfaces or orbifolds should bring new insights in the mathematical theory of the fractional Hall effect.

## B Dynamics and variational methods

### B1 Topological entropy and geodesic flows on surfaces

Bramham, Knieper

1 pd (RUB), 1 ds (RUB)

The focus of this project is to investigate the relation of zero topological entropy and integrability for low-dimensional Hamiltonian systems, notably the geodesic flows on the 2-sphere and the 2-torus, and their generalizations to Reeb flows on the 3-sphere and the 3-torus. For example, using finite energy foliations we would like to prove that Reeb flows on the 3-sphere with a dense orbit and at least three periodic orbits must have positive topological entropy. For low-dimensional Hamiltonian systems with zero topological entropy we hope to show the existence of integrable approximations.

### B2 Minimal geodesics

Knieper, Kunze

1 ds (UzK)

This project investigates minimal geodesics on complete Riemannian manifolds with non-compact universal cover, in particular their ergodic properties w.r.t. naturally defined invariant measures. We shall also deal with twist maps featuring non-periodic angles, which exclude the application of KAM theory or Aubry-Mather theory. However, the dynamics of the minimizing orbits are deeply connected to those described in classic results of Hedlund for the 2-torus. Hence we plan to transfer the

techniques from the non-periodic angles setting to establish structural phenomena of minimal geodesics (like recurrence) on non-compact cylinders, for instance.

### B3 Systolic inequalities in Reeb dynamics

Abbondandolo, Bramham

1 pd (RUB), 1 ds (RUB)

The aim of this project is to extend some notions from classical systolic geometry to Reeb dynamics. In particular, we would like to carry out a thorough study of the ratio of the  $n$ th power of the minimal action of a closed characteristic by the contact volume. Typical questions which we would like to address are: Is this ratio bounded from above on meaningful classes of contact forms? Is the maximum achieved? Although not a symplectically invariant notion, convexity seems to play a special role: why? Answers to these questions are expected to shed some light on important open problems in symplectic topology, Riemannian and Finsler systolic geometry, convex analysis and billiard dynamics.

### B4 Loop groups and the path model

Littelmann, Lytchak

1 ds (UzK)

The project intends to give a geometric reinterpretation of the combinatorial path model in representation theory. The aim is to merge various points of view on the geometric and topological construction of bases of representations of semisimple Lie algebras and their associated combinatorial models. One of these points of view is symplectic; it is based on the study of Mirković-Vilonen polytopes, which characterize one such basis and arise as the image under the momentum map of the relevant basis. Variational methods enter via Morse theory on loop groups.

### B5 Hyperbolicity in dynamics and geometry

Knieper

1 ds (RUB)

This project is concerned with the classification of simply connected harmonic Riemannian manifolds, a problem related to the famous Lichnerowicz conjecture. Besides tools from hyperbolic geometry and dynamics we will study structures arising in symplectic geometry (bipolarized symplectic structures) and complex analysis (Grauert tubes).

### B6 Algebraic and analytic aspects of integrable systems

Burban, Kunze

2 ds (UzK)

The aim of this project is to study completely integrable dynamical systems in finite and infinite dimensions, using techniques from algebraic geometry, homological algebra, Riemann-Hilbert problems and soliton dynamics. The common feature of all those problems is that algebraic or dynamic information can be derived from the knowledge of certain 'spectral objects'. More specifically, we shall

address the Krichever correspondence for ordinary and partial differential operators, Calogero-Moser systems in dimension two, and, on the partial differential equations side, soliton and breather asymptotics for the cubic non-linear Schrödinger equation, which can also include a  $\delta$ -potential on the right-hand side (destroying the complete integrability).

**B7** Lorentz and contact geometry  
Nemirovski, Suhr  
1 ds (RUB)

The aim of the project is to understand the geometry of globally hyperbolic and Zollfrei spacetimes using the natural contact structure on the space of lightlike geodesics. We investigate causality relations using positive Legendrian isotopies, give a new interpretation of redshifts in terms of contact forms, and study the topology of Lorentz manifolds all of whose lightlike geodesics are closed.

## C Algebra, combinatorics and optimization

**C1** Symplectic capacities of polytopes  
Abbondandolo, Albers, Vallentin  
1 ds (RUB), 1 ds (UzK)

The aim of this project is to perform first steps into discrete symplectic geometry by developing and implementing algorithms to compute the Ekeland-Hofer capacity of convex polytopes in a symplectic vector space. These algorithms are based on the dual action principle of Clarke. There are several interesting symplectic and optimization questions connected to this. We expect these algorithms to give us insights into challenging open questions in symplectic topology, billiard dynamics and convex geometry.

**C2** Algorithmic symplectic packing  
Geiges, Jünger, Vallentin  
1 pd (UzK), 1 ds (UzK)

The aim of this project is to develop algorithmic tools for simplex packings that stem from the ball packing problem in symplectic topology. We want to formulate these packing questions as optimization problems so that recent, advanced algorithmic tools from combinatorial optimization (mixed integer non-linear programming, semidefinite optimization) can be used in this context. The insight gained from this experimental work should serve as the basis for further theoretical study. A major challenge, both computationally and theoretically, is the computation of packing widths in dimension greater than four.

C4    Combinatorics of manifolds with symmetries and modularity properties  
Bringmann, Sabatini  
1 pd (UzK)

We use the correspondence between integral polytopes and symplectic toric manifolds to study combinatorial and number-theoretic properties of integral polytopes, in particular reflexive polytopes. This connection allows us to use powerful tools coming from equivariant cohomology and K-theory, as well as the rigidity and modularity properties of equivariant genera on toric varieties. We study the Hilbert and Ehrhart polynomials using this new approach, starting with the investigation of questions posed by Rodriguez-Villegas about the position of their roots.

C5    Modular forms and Gromov-Witten theory  
Bringmann, Zehmisch, Suhr  
1 pd (UzK), 1 pd (WWU)

We will provide a theory of polyfolds and abstract perturbations for holomorphic discs. Applications make it possible to prove instances of the Weinstein conjecture and to answer fillability questions for large classes of contact manifolds. Connections to number-theoretic concepts will be drawn via the study of Gromov-Witten invariants. The aim is to investigate whether generating series of Gromov-Witten invariants are Fourier expansions of certain modular forms. This will help us to detect finer structures of Gromov-Witten invariants for non-Kähler manifolds.